On the interaction of FR-II radio sources with the intracluster medium

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ABSTRACT

The effect of the expansion of powerful FR-II radio sources into a cluster environment is discussed. The analysis considers both the thermal and temporal evolution of the ICM which has passed through the bow shock of the radio source and the effect of this swept-up gas on the dynamics of the radio source itself. The final state of the swept-up ICM is critically dependent on the thermal conductivity of the gas. If the gas behind the bow shock expands adiabatically, and the source is expanding into a steeply falling atmosphere, then a narrow dense layer will form as the radio source lifts gas out of the cluster potential. This layer has a cooling time very much less than that of the gas just ahead of the radio source. This effect does not occur if the thermal conductivity of the gas is high, or if the cluster atmosphere is shallow. The swept-up gas also affects the dynamics of the radio source especially as it slows towards sub-sonic expansion. The preferential accumulation of the swept-up gas to the sides of the cocoon leads to the aspect ratio of the source increasing. Eventually the contact surface must become Rayleigh-Taylor unstable leading both to inflow of the swept-up ICM into the cavity created by the cocoon, but also substantial mixing of the cooler denser swept-up gas with the ambient ICM thereby creating a multi-phase ICM. The radio source is likely to have a marked effect on the cluster on timescales long compared to the age of the source.

Key words: hydrodynamics – galaxies: active – galaxies: jets – X-rays: galaxies : clusters – galaxies: cooling flows

1 INTRODUCTION

Observations with both the ROSAT and CHANDRA satellites have demonstrated clearly that the presence of a powerful FR-II radio source very significantly affects the ICM/IGM in the vicinity of the source. The simplest model for such a radio source has an over-pressured cocoon of jet material expanding supersonically into the external gas; expanding ahead of the cocoon a bow shock sweeps up the surrounding gas which then forms a shell between the bow-shock and the contact surface separating shocked gas and cocoon material. In this model the cocoon should therefore be devoid of ICM/IGM and the strong shock should heat the ICM/IGM. Some observations, but not all, support this general picture. Both ROSAT and CHANDRA data for Cygnus A (Carilli, Perley & Harris 1994; Smith et al. 2002) show cavities in the X-ray emitting gas coincident with the radio cocoon and the effects of the bow shock can be clearly seen. The observations of both the A426/3C84 interaction (Perseus; Fabian et al. 2000; Fabian et al. 2002) and A2052/3C317 show clear evidence for a radio cocoon devoid of X-ray emitting gas although in both cases the radio structure is complex (Rizza et al. 2000; Blanton et al. 2001). In two cluster/radio-source interactions in A133 and

A2626 studied by Rizza et al. (2000) using ROSAT the X-ray emitting gas may well occupy the same volume as the radio-emitting plasma. The interaction of FR-I or complex radio sources with their environment has often been interpreted in terms of the buoyancy effects of the low-density radio-emitting plasma as in M87 (Böhringer et al. 1995; Brügen and Kaiser 2001), Hydra A (McNamara et al. 2000a; David et al. 2001; Nulsen et al. 2002) and Centaurus A (Saxton, Sutherland & Bicknell 2002). A recent review of the X-ray observational data on radio-source/cluster interaction is given in McNamara et al. (2002b). This interaction between radio source and the ICM has led a number of authors to consider whether radio sources could solve the so-called "cooling-flow problem" (e.g. Binney and Tabor 1995; McNamara et al. 2000b; Reynolds, Heinz & Begelman 2001; Churazov et al. 2001; Quilis, Bower & Balogh 2001; Böhringer et al. 2002), in which the cool gas expected to accumulate at the cluster centre as a result of cooling is not detected.

A number of authors have considered various aspects of the physics and evolution of the gas swept-up by the passage of the radio source. Scheuer (1974) pointed out the potential for the swept-up gas to become Rayleigh Taylor unstable given that the density of the cocoon must be very much less than that of the external gas. Gull & Northover (1973) had

already considered the evolution of a light bubble of gas in a cluster potential as a possible model for radio source confinement, while Smith et al. (1983) determined the conditions under which a bubble, rather than a jet, would be produced in the Blandford and Rees (1974) twin-exhaust model. This picture has recently been updated by Churazov et al. (2001), Brüggen & Kaiser (2001), Quillis, Bower & Balogh (2001) and Saxton, Sutherland & Bicknell (2002) who have investigated the evolution of buoyant bubbles. These studies show that the buoyant material can give rise to a mushroom-type structure which although susceptible to Rayleigh-Taylor instability can significantly affect a cooling flow within the cluster. Heinz, Reynolds & Begelman (1998; hereafter HRB), calculated the expected X-ray emission from the swept-up ICM using a simplified geometry for the radio source and assuming uniform conditions between the bow-shock and the contact surface. A more detailed calculation of the heating effects of a radio source on its environment was made by Kaiser & Alexander (1999; hereafter KA99) assuming an elliptical source shape, self-similar evolution and adiabatic conditions in the swept-up gas. They predict significant structure in the X-ray emission behind the bow shock and flow in this region away from the hotspot. Reynolds, Heinz & Begelman (2001) present detailed simulations of a radio source expanding into an atmosphere; for sources expanding supersonically their results are in agreement with the analytical models of KA99, but they are able to follow the evolution of the radio source into a phase in which it reaches pressure balance with the surrounding gas and again find significant structure in the expected X-ray emission. Despite this literature a number of important problems concerning the evolution of the swept-up gas have not been fully addressed. In particular what is the long term effect of the radio source on its environment on time-scales long compared to the source life time, and what effect does the swept-up gas have on the evolution of the radio source. Heating of the ICM/IGM by radio sources may have important and measurable cosmological effects (e.g. Yamada & Fujita 2001).

In this paper I extend this earlier work to determine the effects of the expansion of a powerful FR-II radio source into a cluster-like environment. The structure of this paper is as follows. In Section 2 I outline the main physical considerations appropriate to the analysis of the evolution of gas swept-up by the radio source. In Section 3 I consider the evolution of this gas in detail and in Section 4 I consider the effects of the gas on the dynamics of the radio source. Finally, in Section 5, I discuss the implications of these results.

2   GENERAL CONSIDERATIONS

Before discussing more detailed models, it will be useful to consider the main factors which are likely to be important for the swept-up gas and the radio source itself. I begin by reviewing the likely physical conditions in the swept-up gas. Figure 1 shows a sketch of the assumed geometry of the source. For simplicity I shall refer to the atmosphere into which the radio source propagates as the Intra Cluster Medium (ICM).

As the radio source expands it must do work on the sur-

rounding medium. For supersonic expansion the ICM will be heated by the bow shock and is also lifted out of the potential well in which the host galaxy sits; this swept-up gas resides between the bow shock and contact surface. The sound speed in this shocked gas must be of order the velocity of the bow shock itself, and therefore the sound crossing time must be somewhat less than the age of the radio source suggesting that the pressure between the bow shock and contact surface should be approximately constant. This argument is confirmed by numerical simulations of FR-II radio sources (e.g. Reynolds et al. 2001) and the calculations of KA99. There must be pressure variations within the shocked gas around the periphery of the source since the pressure at the hotspot exceeds that within the cocoon; these pressure variations drive a flow within the swept-up gas sending material away from the hotspot in the direction of the host galaxy (KA99, Reynolds et al. 2001).

As the ICM passes through the bow shock it is of course heated. As this gas flows downstream of the bow shock it will evolve adiabatically unless there is significant radiative heating / cooling or the thermal conductivity is high. The issue of the cooling of the gas will be considered in detail below. The X-ray gas is optically thin, therefore it is only necessary to consider the thermal conductivity as a heating source. The thermal conductivity can be written in the form $1.3\eta n_e \lambda_e (kT_e/m_e)^{1/2}$, where the subscript e refers to electron values, λ_e is the electron mean free path, and the dimensionless factor η allows for suppression of the thermal conductivity below the canonical form given by Cowie & McKee (1977). The heating time, t_h, for a temperature variation of ΔT to be removed in a layer of gas of thickness Δr is given by $t_h \approx 1.5 n k \Delta T \Delta r / q$, where the heat flux, $q = 1.3\eta n_e \lambda_e (kT_e/m_e)^{1/2} \Delta(kT)/\Delta r$. This simplifies since $(kT_e/m_e)^{1/2} \sim \bar{v}_e \sim c_s (m_p/m_e)^{1/2}$, hence $t_h \sim 0.03 t_s (\Delta r/\lambda_e)/\eta$, where t_s is the sound crossing time across the region of size Δr. The electron mean free path is given approximately by:

$$\lambda_e \approx 23 \left(\frac{T_e}{10^8 \mathrm{K}}\right)^2 \left(\frac{n_e}{10^{-3}\mathrm{cm}^{-3}}\right)^{-1} \mathrm{kpc}.$$

For Cygnus A at a radius of approximately 100 kpc $n_e \sim 7 \times 10^{-3}\mathrm{cm}^{-3}$, $T_e \sim T_{\mathrm{gas}} \sim 7 \times 10^7 \mathrm{K}$, $\Delta r \sim 10\mathrm{kpc}$ (Smith et al. 2002), and therefore

$$\frac{t_h}{t_s} \approx \frac{0.3}{\eta}.$$

For the heating time of order the sound crossing time, and both much less than the age of the source, the swept-up layer of gas should be both isothermal and isobaric. However, if the thermal conductivity is significantly suppressed over the canonical form given by Cowie & McKee then the gas will evolve adiabatically behind the shock. Recently, Ettori & Fabian (2000) have argued, based on the analysis of the X-ray data for Abell 2142, that the thermal conductivity is very significantly suppressed giving $\eta \sim 1/250$ to as low as $\sim 1/2500$ at least in some regions of the cluster. If the suppression of the thermal conductivity is of this order in all cases then the gas behind the shock should clearly be treated adiabatically. Given the remaining uncertainty in the value for the thermal conductivity, I shall discuss both the adiabatic and isothermal cases below. The temperature of the swept-up gas will be determined by dynamical pro-
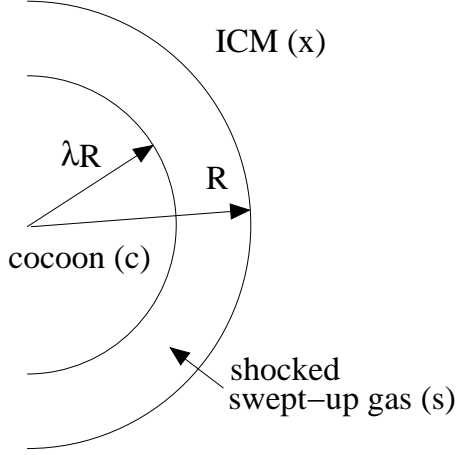
Figure 1. The simplified geometry assumed for models of the cocoon expansion

cesses (e.g. shock heating, adiabatic expansion) and radiative cooling. I will refer to the former processes as dynamical heating/cooling.

The mass of swept-up gas affects the dynamics of the radio source in two ways: firstly the inertia of the gas must be considered and secondly the gravitational force on the gas due to the cluster potential. As I will show below the inertia of the gas does not alter the dynamics in the sense that the radio source may expand in a self-similar fashion as shown by Kaiser & Alexander (1997; hereafter KA97) – their model implicitly allows for the inertia of the gas. A simple calculation demonstrates that the gravitational force of the swept up gas will also be important towards the end of the life time of the source. As the source expansion slows the cocoon pressure drops towards pressure equilibrium with the confining atmosphere. For gas in a dark matter dominated cluster with no pre-heating of the ICM, the pressure in the gas is simply determined by the requirement for hydrostatic equilibrium in the potential well, hence the pressure in the gas must be comparable to the gravitational potential energy per unit volume of gas at any point in the cluster. This gravitational force must therefore be at least as important as the pressure in the atmosphere as the source moves towards sub-sonic expansion.

Throughout this paper I will use the following conventions: subscripts j, c, h, s will refer to the jet, cocoon, hotspot and swept-up gas respectively; the parameters in the external gas will be denoted by the subscript x. The external atmosphere is assumed to follow a King profile of the form

$$\rho_x = \left(1 + (r/a_0)^2\right)^{-3\beta_K/2}$$

which is approximated as a piece-wise power-law (Alexander 2000), $\rho_x = \rho_0(r/a_0)^{-\beta}$, with $\beta \sim 0$ for $r < a_0$ and $\beta = 3\beta_K$ for $r \gg a_0$. Finally, I assume adiabatic indices for the ICM of 5/3 and for the radio-emitting plasma of the cocoon of 4/3.

3   EVOLUTION OF THE SWEPT-UP GAS

The discussion of Section 2 shows that it is the expansion of the cocoon which is most important in determining the

evolution of the swept-up gas. A simplified model is then sufficient to investigate this evolution and derive scaling relationships and I therefore approximate the expansion of the cocoon as a spherical expansion following the treatment by HRB.

The modelled geometry of the cocoon and bow shock is shown in Figure 1. The radius of the bow-shock is R and that of the cocoon (or equivalently the contact surface) λR. The dynamics of the cocoon are determined by the energy input from the jet at a constant rate Q_0. As shown in KA97 and by HRB for this simplified geometry (see also Section 4.2) the bow-shock and cocoon expand self-similarly with λ a constant and the radius of the bow shock given by $R = Ca_0 \left(\frac{t}{t_0}\right)^{\frac{3}{5-\beta}}$, where the characteristic time $t_0 = \left(a_0^5 \rho_0 / Q\right)^{1/3}$.

Two related parameters which characterise the nature of the swept-up gas are: (1) the ratio of the cooling time in this gas compared to that of the ICM just ahead of the bow shock, and (2) the ratio of volume emissivities due to thermal Bremsstrahlung for the swept-up gas and un-shocked ICM. The former depends on the bolometric luminosity and therefore, for gas of order a few 10^7K, scales as $T^{1/2}/\rho$, whereas the appropriate emissivity for a relatively narrow band detector is the specific emissivity and therefore scales approximately as $\rho^2 T^{-1/2}$.

3.1   The isothermal case

I consider first the case when the gas between the bow-shock and contact surface is taken to be isothermal; this assumption was used by HRB who then calculated the X-ray emissivity for a spherical source expanding in a King profile. Since the pressure is approximately constant through the swept-up gas (Section 2) the density, ρ_s, is uniform and can be found simply from the total swept-up mass, $4\pi R^{3-\beta}\rho_0 a_0^\beta/(3-\beta)$, and the volume between bow shock and contact surface, $4\pi(1-\lambda^3)R^3/3$. The pressure in the swept-up gas follows from the jump conditions at the shock which I shall assume to be a strong shock, $p_s = \frac{5}{4}M_0^2 p_x = \frac{3}{4}\rho_x \dot{R}^2$, where a dot denotes differentiation with respect to time, $\rho_x = \rho_0(R/a_0)^{-\beta}$ is the density just ahead of the shock at a radius R, and M_0 is the Mach number of the bow-shock relative to the ICM for a source of radius R. The temperature in the swept-up gas, T_s, follows from the ideal gas law, $p = \rho k T/\mu$, where μ is the mass per particle:

$$T_s = \frac{\mu p_s}{k\rho_s} = \frac{5}{4}M_0^2 \frac{\rho_x}{\rho_s}T_x.$$

The cooling time of both the swept-up gas and the ICM just ahead of the shock are proportional to $T^{1/2}/\rho$, hence the ratio of the cooling time in the swept-up gas, t_{cs} to that of the ICM just ahead of the shock, t_{cx} is

$$\frac{t_{cs}}{t_{cx}} = \frac{T_s^{1/2}}{\rho_s}\frac{\rho_x}{T_x^{1/2}} = \frac{\sqrt{5}}{2}M_0\left(\frac{\rho_x}{\rho_s}\right)^{3/2}.$$

The density in the swept-up gas is given by $\rho_s = \frac{3}{(3-\beta)(1-\lambda^3)}\rho_x$, hence:

$$\frac{t_{cs}}{t_{cx}} = \frac{\sqrt{5}}{2}\left((3-\beta)(1-\lambda^3)\right)^{3/2}M_0.$$
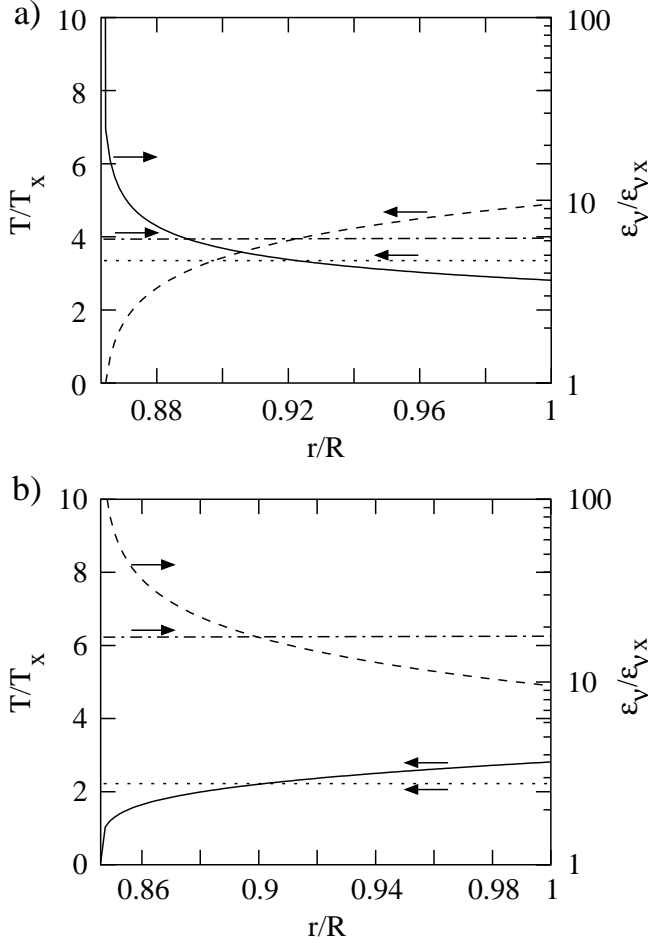
a)

b)

Figure 2. The radial dependence of the temperature and volume emissivity in the region between the contact surface and the bow shock. The two panels show the results for two different atmospheres (a) $\beta = 0.5$ and (b) $\beta = 1.5$. In both (a) and (b) the lines have the following meaning: solid line T/T_x adiabatic; dotted line T/T_x isothermal; dashed line ϵ_v/ϵ_{vx} adiabatic; dashed-dot line ϵ_v/ϵ_{vx} isothermal

$(1 - \lambda^3)$ is equal to $15/4(11 - \beta)$ (Section 4.2), hence $\frac{t_{cs}}{t_{cx}} > 0.05 M_0$ since $\beta < 2$. Except for high Mach numbers the cooling time in the swept-up gas is less than that in the external atmosphere ahead of the shock, and this will certainly be the case as the source slows to near sub-sonic expansion.

Similarly, the ratio of specific volume emissivities is given by:

$$\frac{\epsilon_{vs}}{\epsilon_{vx}} = \frac{T_s^{-1/2} \rho_s^2}{T_x^{-1/2} \rho_x^2} = \frac{2}{\sqrt{5}} \left(\frac{4}{5} \frac{3}{(3-\beta)(1-\lambda^3)} \right)^{5/2} M_0^{-1},$$

and $\frac{\epsilon_{vs}}{\epsilon_{vx}} < 125 M_0^{-1}$. These results are illustrated in Figure 2.

The enhanced emissivity of the swept-up gas over the ICM ahead of the shock leads to an X-ray bright shocked shell as discussed by HRB. The mean isothermal temperature in the shocked gas differs slightly from the post-shock temperature by a factor $5(3-\beta)/(11-\beta)$ in this approximation, however even in a steep atmosphere the factor is close to unity, hence the swept-up gas never cools due to dynam-

ical cooling significantly below the temperature of the gas ahead of the shock. In the later stages of expansion as the Mach number tends to unity, the cooling time can become short compared to that in the cluster gas – if the timescale is comparable to the age of the source then significant radiative cooling of the swept-up gas can occur. Taking a source age of 10^7 yrs and a steep atmosphere, then the cooling time in the cluster gas would have to be less than of order a few 10^8 yrs for the swept-up gas to suffer significant radiative losses. Such conditions are only possible in the centre of the strongest cooling flow clusters. For sources still within the core radius of the cluster the cooling time in the ICM would have to be nearly an order of magnitude less. It seems likely therefore that if the gas is isothermal radiative cooling of the swept-up gas is negligible over the lifetime of the source.

3.2   The adiabatic case

The situation is very different if the thermal conductivity is sufficiently suppressed so that the swept-up gas evolves adiabatically. Consider gas which passed through the shock when the bow-shock was of radius Ry, where R is the radius of the bow shock at the current epoch and $0 \le y \le 1$. The immediate post-shock conditions result in a density and temperature (ρ_y, T_y) which are simply related by the jump conditions to the density and temperature in the ICM; the density across the shock rises by a factor of 4, and the temperature increases by a factor of $\frac{5}{16} M_y^2$, where M_y is the Mach number of the shock at a radius Ry. As the source evolves to a size R this gas must expand and will cool adiabatically. The density and temperature of this gas when the source reaches a size R $(\rho_s(y), T_s(y))$ are given by the usual relationships

$$\frac{\rho_s(y)}{\rho_y} = \left(\frac{p_s}{p_y} \right)^{1/\gamma}$$

$$\frac{T_s(y)}{T_y} = \left(\frac{p_s}{p_y} \right)^{1-1/\gamma}$$

where $\frac{p_s}{p_y}$ is the ratio of the post shock pressure for cocoon radii of R and Ry. In the self-similar model of KA97 (see HRB; and Section 4.2) this pressure ratio has a power-law dependence on the source size of

$$\frac{p_s}{p_y} = y^{\frac{4+\beta}{3}}.$$

Assuming an isothermal atmosphere, the Mach number at radius Ry is related in the self-similar model to the Mach number at radius R by $M_y = M_0 y^{(\beta-2)/3}$. Using these results it follows that the density and temperature at the present epoch for gas swept up when the source was of size Ry are given in terms of the external conditions at a radius R as follows:

$$\rho_s(y) = 4\rho_x y^{\frac{4(1-\beta)}{5}}$$

$$T_s(y) = \frac{5}{16} T_x M_0^2 y^{\frac{4(\beta-1)}{5}}.$$

The distribution of density and temperature between the contact surface and bow shock at the present epoch follow from conservation of mass. The mass swept up between radii Ry to $R(y+dy)$ must now lie between radii Rx to $R(x+dx)$ such that:

$$4\pi x^2 dx R^3 \rho_s(y) = 4\pi y^2 dy R^3 \rho_x y^{-\beta}.$$

Substituting the above results for the density gives a simple differential equation relating x and y which has the solution

$$x^3 = \frac{15 y^{(11-\beta)/5}}{4(11-\beta)} + \lambda^3$$

which is valid for $\lambda \leq x \leq 1$ and $\lambda^3 = 1 - 15/4(11 - \beta)$. These solutions are illustrated in Figure 2.

These results are in agreement with the numerical calculations of HA99 and the adiabatic wind models developed by Dyson, Falle & Perry (1980). It follows that the cooling time of this gas compared to the cooling time of the gas immediately ahead of the shock at radius R is given by

$$\frac{t_{cs}(y)}{t_{cx}} = \frac{\sqrt{5}}{16} M_0 y^{\frac{6(\beta-1)}{5}}.$$

For $\beta \leq 1$ the cooling time of the post-shock gas will, for even very modest Mach numbers, exceed that of the gas at a similar radius in the ICM. For $\beta > 1$ some fraction of the gas will have a cooling time less than the ICM at that same radius. For example, taking $\beta = 1.5$, $M_0 = 10$, all gas swept up when $y < 0.54$ will have a cooling time less than the ICM at a radius R, or approximately 40% of the swept-up gas by mass.

The ratio of the specific volume emissivities follows similarly:

$$\frac{\epsilon_{vs}(y)}{\epsilon_{vx}} = \frac{T_s(y)^{-1/2} \rho_s(y)^2}{T_x^{-1/2} \rho_x^2} = \frac{64}{\sqrt{5}} y^{2(1-\beta)} M_0^{-1}.$$

Unlike the isothermal case these results depend critically on the form of the atmosphere into which the source is expanding and when the gas was swept up. Immediately behind the bow shock the X-ray emissivity will be enhanced (unless the expansion is highly supersonic). Between the bow-shock and the contact surface the emissivity will fall for $\beta < 1$, whereas for $\beta > 1$ the emissivity rises towards the contact surface, the temperature falls and the cooling time decreases. For a steep atmosphere we therefore expect to find a layer of cooler, dense X-ray luminous gas adjacent to the contact surface.

4 EFFECTS OF THE SWEPT-UP GAS ON RADIO SOURCE DYNAMICS

I have already discussed in Section 2 how the gravitational force of the swept-up gas must become comparable to the cocoon pressure as the expansion of the radio source slows to being only mildly supersonic. In this section I consider in more detail the evolution of the radio source itself. There are three aspects that must be considered. As first pointed out by Scheuer (1974), the contact surface is liable to Rayleigh Taylor instability unless the deceleration of the radio source is sufficient to stabilise this interface. I shall argue in Section 4.1 that where observations suggest that the X-ray emitting gas is excluded from the cocoon this stability requirement may place a useful additional constraint on the dynamics of the radio source. Even before the source approaches pressure equilibrium the swept-up gas modifies the dynamics of the radio source principally by leading to a departure from self-similar evolution since the swept-up gas accumulates preferentially around the cocoon and not the hotspot. In Section

4.2 I develop a simple dynamical model for this phase of the radio source evolution. Finally in Section 4.3 I consider the later stages of evolution when the source expansion becomes only mildly supersonic.

4.1 Rayleigh-Taylor stability of the contact surface

The contact surface between cocoon and the swept-up gas is liable to become Rayleigh-Taylor (RT) unstable since the density of the swept up material exceeds that within the cocoon. The surface is stabilised by the deceleration of the contact surface (Scheuer 1974). If the total mass of gas and dark matter within a radius r is $M(r)$, and assuming spherical symmetry for the cluster, then the component of the gravitational acceleration perpendicular to the contact surface at a radius r from the centre of the cluster is $g_\perp = GM(r) \cos(\psi)/r^2$, where ψ is the angle between radius vector and the normal to the cocoon. The deceleration of the contact surface cannot be measured directly from observations, but can estimated in a (weakly) model dependent manner by assuming a scaling relation for the advance of the contact surface of the form $R_c = \alpha t^\delta$, where R_c is an appropriate measure of the size of the cocoon. It follows that the acceleration of the contact surface can be written:

$$\ddot{R}_c = \delta^3(\delta - 1) \frac{\dot{R}_c^2}{R_c}.$$

For the self-similar model of KA97 the exponent $\delta = 3/(5 - \beta)$, and R_c and $\dot{R}_c$, can both be estimated from observational data; for the advance of the hotspot R_c is simply the length of the source, or for the cocoon it is the radius of the cocoon assuming approximately cylindrical geometry.

Observations with CHANDRA show the cocoons of a number of radio sources, in particular Cygnus A, to be devoid of X-ray emitting gas. For the contact surface to exclude the swept-up gas the surface must remain stable to RT instabilities otherwise fast-growing short wavelength modes would disrupt the contact surface allowing the swept-up mass to fill the cocoon cavity. The requirement for stability at any point on the contact surface requires the surface to be decelerating and:

$$|g_\perp| < |\ddot{R}_c| = |\delta^3(\delta - 1) \frac{\dot{R}_c^2}{R_c}|.$$

Smith et al. (2002) give data for the total mass (dark matter and gas) within a radius r in the Cygnus A cluster. Using these results the perpendicular component of the gravitational acceleration at the hotspot of Cygnus A is approximately $7.4 \times 10^{-10} \mathrm{ms}^{-2}$, and for a point on the cocoon approximately three-quarters along the lobe $1.4 \times 10^{-10} \mathrm{ms}^{-2}$. For self-similar models the magnitude of the deceleration decreases as the atmosphere becomes steeper. To obtain an upper estimate for the deceleration I therefore adopt a lower limit to β of ~ 1.5 (Carilli et al. 1994; Smith et al 2002), giving $\ddot{R}_c = 0.09 \dot{R}_c^2/R_c$. The hotspot advance speed can be estimated from spectral ageing arguments assuming an equipartition field strength within the lobes. This gives for Cygnus A $\dot{L}_j \sim 0.05c$ (Alexander Brown & Scott 1984) and since $L_j \sim 100 \mathrm{kpc}$

$$|\ddot{L}_j| = 6.75 \times 10^{-9} \left(\frac{v}{0.05c}\right)^2 \left(\frac{L_j}{100\text{kpc}}\right)^{-1} \text{ms}^{-2}.$$

For self-similar expansion the acceleration of the side of the cocoon is related to the hotspot acceleration via $\ddot{r}_c = \frac{r_c}{L_j}\ddot{L}_j$, giving for the cocoon $\ddot{r}_c \sim 0.25\ddot{L}_j$. The lobe speed derived from spectral ageing is almost certainly an upper limit. Taking into account all of the dynamical constraints Alexander & Pooley (1996) estimate $\dot{L}_j \sim 0.005c$. At best therefore the contact surface at the hotspot of Cygnus A is likely to be just RT stable. I will return to this point in Section 5.

In principle, this method provides a way of obtaining a lower-limit to the advance speed of a radio source when there is evidence that the contact surface is still RT stable; the method relies only upon the existence of good X-ray data and a simple, but robust, model for the source expansion.

4.2   Dynamical model with swept-up gas

In this section I investigate whether the swept-up mass has any effect on the dynamics of the source before the source ceases to expand supersonically. KA99 show that there is significant flow in the swept up gas driven by the pressure gradient between the hotspot and the sides of the cocoon. A simplified model for the dynamics is therefore suggested in which there is no significant effect at the hotspot and only the dynamics of the lobe are affected by the gravitational force on the swept-up gas in the cluster potential. In order to make progress I therefore adopt a simplified approach in which I treat the expansion of the cocoon as purely spherical; this is the approach taken by HRB and Reynolds & Begelman (1997). The expansion of the source in the hotspot region will be similar to that discussed by Falle (1991) and KA97, and the two parts of the overall solution can be related by requiring that the jet is in pressure balance with the cocoon as in KA97.

The governing equations are similar to those given by HRB, although I now allow for both the inertia and gravitational force on the swept-up mass and use a more accurate treatment of the jump conditions at the shock. The dynamical analysis assumes the gas behaves adiabatically throughout. As before, the radius of the bow shock is defined as R and the radius of the cocoon is written as $R_c = \lambda R$. The first equation gives the conservation of energy in the cocoon assuming that negligible energy is stored in the hotspot:

$$\frac{1}{\gamma_c - 1}V_c\frac{dp_c}{dt} + \frac{\gamma_c}{\gamma_c - 1}p_c\frac{dV_c}{dt} = Q_0.$$

Assuming a uniform pressure in the swept-up layer, energy conservation in this gas gives

$$\frac{1}{\gamma_s - 1}V_s\frac{dp_s}{dt} + \frac{\gamma_s}{\gamma_s - 1}p_s\frac{dV_s}{dt} = \frac{15}{16}\frac{4\pi R^2}{2}\rho_0\left(\frac{R}{a_0}\right)^{-\beta}\dot{R}^3$$

where the term on the right hand side is just the rate of heat input at the bow shock (the factor of 15/16, which differs from HRB, allows for the bulk kinetic energy of the gas post-shock). The approximation of a uniform pressure between bow shock and contact surface is not exact when the inertia of the swept-up gas is not negligible. To simplify the problem I assume, given the results of Section 3, that most of the mass resides in a relatively thin layer either at the contact surface when $\beta > 1$, or just behind the bow

shock when $\beta < 1$. Across the rest of the region between bow shock and contact surface I assume a uniform pressure of p_s. The pressure difference between cocoon and the shocked gas can be equated to the acceleration of this narrow dense layer plus the gravitational acceleration. For $\beta > 1$

$$4\pi\lambda^2 R^2(p_c - p_s) = M_s\left((\ddot{\lambda R}) + g_D(\lambda R)\right)$$

Where $M_s = \frac{\rho_0 a_0^\beta}{3-\beta}R^{3-\beta}$ is the swept-up mass, and $g_D(r)$ is the gravitational acceleration towards the centre of the cluster at radius r, due mainly to the dark matter in the cluster. When $\beta < 1$ this equation is modified in the sense that λR is everywhere replaced by R. This equation differs from the equivalent form given by HRB by inclusion of both an inertia term and the gravitational acceleration; as outlined below the inclusion of the inertia term still permits a self-similar solution to be found when the gravitational term is negligible (KA97 implicitly allow for the inertia of the swept-up gas in their dynamical solution).

The gravitational acceleration is found in a self-consistent manner by assuming that the distribution of dark matter is such as to maintain the gas in hydrostatic equilibrium; for an isothermal gas distribution it follows that the gravitational acceleration is given by:

$$g_D = \frac{kT_x}{\mu}\frac{1}{\rho_x}\frac{d\rho_x}{dr},$$

and for a King profile

$$g_D = 3\beta_K\frac{kT_x}{a_0\mu}\frac{r}{a_0}\frac{1}{\left[1 + (r/a_0)^2\right]}$$

which can be approximated as follows:

$$g_D \approx X_D \begin{cases} \left(\frac{r}{a_0}\right)^{-1} & r \gg a_0 \\ \left(\frac{r}{a_0}\right) & r \ll a_0 \end{cases}$$

where $X_D = 3kT_x\beta_K/\mu a_0$.

The final relationships needed to close this set of equations are $V_c = 4\pi\lambda^3 R^3/3$, $V_s = 4\pi(1 - \lambda^3)R^3/3$, and $p_s = \frac{3}{4}\rho_0\left(\frac{R}{a_0}\right)^{-\beta}\dot{R}^2$, which follows from assuming the bow shock is a strong shock. In this expressions and throughout I will assume $\gamma_s = 5/3$ and $\gamma_c = 4/3$.

These equations can now be cast in dimensionless form by writing $t = t_0\tau$, $R = a_0 f(\tau)$, where f and τ are the dimensionless bow shock radius and time respectively. Further, I shall write $p_c = \zeta(\tau)p_s$ and note that given the constants of the problem (Q_0, a_0 and ρ_0) the characteristic time $t_0 = \left(\frac{a_0^5\rho_0}{Q_0}\right)^{1/3}$. In terms of the dimensionless parameters the three governing equations become:

$$3\lambda^3 f^3\frac{d}{d\tau}\left(\zeta\dot{f}^2 f^{-\beta}\right) + 4\zeta\dot{f}^2 f^{-\beta}\frac{d}{d\tau}\left(\lambda^3 f^3\right) = \frac{1}{\pi}$$

$$\frac{3}{2}(1 - \lambda^3)f^3\frac{d}{d\tau}\left(\dot{f}^2 f^{-\beta}\right) + \frac{5}{2}\dot{f}^2 f^{-\beta}\frac{d}{d\tau}\left((1 - \lambda^3)f^3\right) = $$
$$\frac{15}{8}f^{2-\beta}\dot{f}^3$$

$$\frac{3(3-\beta)}{4}\lambda^2 f^{2-\beta}\dot{f}^2 = f^{3-\beta}\left(\frac{d^2}{d\tau^2}(\lambda f) + Y_D(\lambda f)^{\beta_D}\right)$$

where $Y_D = t_0^2 X_D/a_0$, $\beta_D = \pm 1$ and a dot indicates differentiation with respect to τ. Again the third equation is written

for $\beta > 1$; for $\beta < 1$ there is no λ dependence. These equations admit a self-similar solution when the gravitational term is negligible $Y_D \approx 0$ with λ and ζ equal to constant values (λ_0, ζ_0) and $f = C\tau^\delta$. Substituting for this form for f gives, after a little algebra, $(1 - \lambda_0^3) = \frac{15}{4(11-\beta)}$, $\delta = \frac{3}{5-\beta}$, and $\zeta_0 = 1 - \frac{1-\delta}{\delta}\frac{4}{3(3-\beta)\lambda_0}$ for $\beta > 1$ and a similar expression omitting the λ_0 for $\beta < 1$. The results are in agreement with the results of KA97 and the previous section.

To investigate the case when Y_D is small but non-zero, I seek a perturbed solution of the form $f = f_0(\tau)(1 + F)$, $\lambda^3 = \lambda_0^3(1+L)$ and $\zeta = \zeta_0(1+Z)$, where $f_0(\tau) = C\tau^{3/(5-\beta)}$, the self-similar solution. These trial solutions are then substituted into the original equations and any terms which are quadratic or higher powers of F, L, Z, Y_D or any differential of one of these terms with respect to τ are neglected. The remaining equations are then linear combinations of $F, L, Z, \tau\dot{F}, \tau^2\ddot{F}, \tau\dot{L}$ and $\tau^2\ddot{L}$, together with the gravitational term which has the form $\tau^2 Y_D(\lambda_0 f_0)^{\beta_D-1}$. A solution then exists such that each term (F, L, Z) has a power-law dependence on dimensionless time of the form τ^α, and each term has the same time dependence as the perturbation, $\tau^2 Y_D(\lambda_0 f_0)^{\beta_D-1}$, giving $\alpha = 2 + 3(\beta_D - 1)/(5 - \beta)$. Substituting this form for the perturbation into the original equations results in a set of 3 coupled linear algebraic equations which can be solved to give the magnitudes (F_1, L_1, Z_1) of each term (e.g. $F = F_1\tau^\alpha$) in terms of the perturbation Y_D. Further details are given in the appendix. For large cluster radii, $\beta_D = -1$ and if the atmosphere is steep $\beta \approx 2$, $\alpha \approx 0$ and we recover a self-similar solution since the perturbation does not grow with time.

These solutions can be used to investigate how the aspect ratio of the source, which I define here to be given by the ratio of the jet to the radius of the cocoon, L_j/R_c, changes with time. The time dependence of R_c in terms of the perturbed solution is straightforward $R_c/a_0 = \lambda f = \lambda_0 f_0(1 + F + L/3)$ to first order. To determine the time dependence of L_j in this approximation we return to the method employed by KA97 where the jet as it propagates through the cocoon is assumed to be confined by the cocoon pressure, $p_j = p_c$. KA97 show that this implies that the hotspot pressure and cocoon pressure must have the same time dependence; this result still holds in the current approximation.

Assuming no accumulation of material at the hotspot, the advance of the hotspot is simply determined by balancing the ram pressure to the hotspot pressure:

$$\rho_0 \left(\frac{L_j}{a_0}\right)^{-\beta} \dot{L_j}^2 = p_h \propto p_c$$

where the time dependence of p_c must be given by the perturbed solution. For an approximately power-law dependence of L_j on time, $\dot{L_j} \sim L_j/(t_0\tau)$. The cocoon pressure depends on $\zeta f^{-\beta}\dot{f}^2$, hence writing the time dependence to first order

$$L_j^{(2-\beta)\delta} \propto \tau^{(2-\beta)\delta-2}\left(1 + (2-\beta)F + \frac{2\alpha}{\delta}F + Z\right)$$

and $R_c \propto \tau^\delta(1 + F + L/3)$. In Figure 3 I show how L_j and R_c evolve with time under the perturbation. The cocoon radius changes slowly compared to the length of the jet; the cocoon pressure in the perturbed solution exceeds that in the self-similar case at the same time since there must now
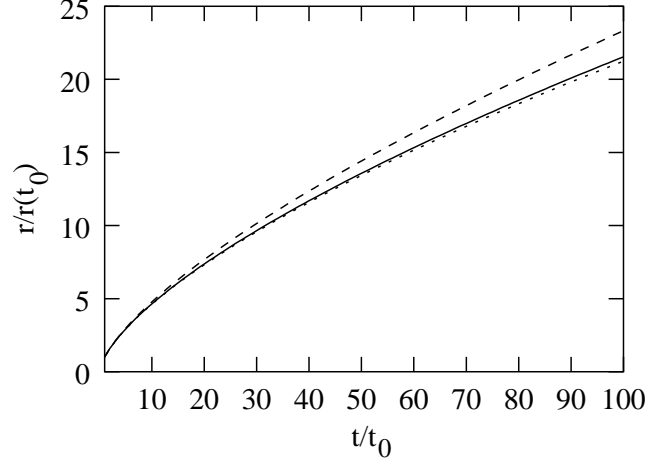

Figure 3. The evolution of the jet length and cocoon radius for the perturbed solution discussed in the text compared to the unperturbed solution. The curves are normalized to the unperturbed solution at $t = t_0$.

be a greater pressure difference between shocked gas and the cocoon to overcome the gravitational force on the swept-up gas and the cocoon volume is correspondingly smaller. This increased lobe pressure acts to collimate the jet since the jet is assumed to be in pressure balance with the cocoon material and the jet therefore propagates more quickly through the external atmosphere. The evolution of the aspect ratio of the source from its unperturbed (similarity) value follows from the above considerations and is given by:

$$\frac{L_j}{R_c} = \left(\frac{L_j}{R_c}\right)_0 \left(1 - \frac{L}{3} + \frac{2\alpha F}{\delta(2-\beta)} + \frac{Z}{2-\beta}\right).$$

The perturbation term is proportional to $Y_D\tau^\alpha$, and the associated coefficient is found by solving the algebraic equations as given in the appendix. I now consider these results for a typical case when $\beta_K = 0.5$, and in the two limits when $L_j \ll a_0$ i.e. $\beta = 0$ and $\beta_D = 1$, and $L_j \gg a_0$ i.e. $\beta = 3/2$ and $\beta_D = -1$:

$$\frac{L_j}{R_c} \approx \left(\frac{L_j}{R_c}\right)_0 \times \begin{cases} 1 + 0.2Y_D\tau^2 & L_j \ll a_0 \\ 1 + 2.9Y_D\tau^{2/7} & L_j \gg a_0 \end{cases}$$

These results can be cast in a more useful form as follows. The perturbation term has the form $Y_D = \frac{3}{4}\frac{kT_x t_0^2}{\mu a_0^2}$, which is equal to $\frac{9}{20}\frac{c_{sx}^2 t_0^2}{a_0^2}$, where c_{sx} is the (constant) sound speed in the ICM. Furthermore, since $\dot{L_j} = \frac{3}{5-\beta}a_0 C t^{\frac{\beta-2}{5-\beta}}t_0^{\frac{-3}{5-\beta}}$, a_0 can be re-written in terms of $\dot{L_j}$ to give:

$$\frac{L_j}{R_c} \approx \left(\frac{L_j}{R_c}\right)_0 \times \begin{cases} 1 + \frac{0.04}{M^2}\left(\frac{t}{t_0}\right)^{6/5} & L_j \ll a_0 \\ 1 + \frac{0.9}{M^2} & L_j \gg a_0 \end{cases}$$

In this expression M is the Mach number of the hotspot advance at the time t. Although the perturbation grows faster for $L_j < a_0$, the effect is never significant since the source reaches the core radius at a time $t \sim t_0$. For $L_j \gg a_0$ the perturbation can be significant especially as the source slows to being only mildly supersonic. The perturbation grows most slowly for steep atmospheres (as expected), however for the example considered here even for $\beta_K = 0.5$ the gravitational perturbation can significantly change the aspect ratio of the

source in the sense that older sources will tend to be longer and thinner. Of course, the considerations of the previous section show that at some point the contact surface must become RT unstable, and hence it is unlikely that in the majority of cases very long thin sources can be formed by this mechanism alone.

4.3   The final stages of the cocoon

The latter stages of development of the cocoon are the most difficult to treat with simple analytical arguments. The gravitational force due to the swept-up gas becomes comparable to the ram pressure as the source slows to being mildly supersonic.

Of particular interest is the ultimate fate of the swept-up gas. One possibility is that as the source slows the onset of the RT instability (Section 4.1) leads to this gas falling back through the contact surface and refilling the low-density region previously occupied by the cocoon. Brügen & Kaiser (2001) have performed simulations to model the latter stages of evolution of a radio source by considering of a light bubble of plasma initially in pressure balance with the gas in the cluster. They find very different behaviour depending on the initial shape of the cloud. Near spherical clouds are quickly destroyed by the RT instability leading to mixing of the cocoon and ICM. Elongated clouds can form a mushroom-type feature which rises through the potential of the cluster in a manner first suggested by Gull and Northover (1973). In the wake behind these mushrooms colder material is lifted out of the cluster. In their simulations Brügen & Kaiser do not include the swept-up gas layer I have discussed in this paper. For the isothermal case of Section 3 when the source reaches pressure balance this layer contains mostly gas with a shorter cooling time than the ambient gas in the cluster. The buoyant phase studied by Brügen & Kaiser will then be very efficient at mixing this gas into the ambient material at large cluster radii.

Further discussion of the later stages of the source are beyond the scope of the present paper. We are currently undertaking simulations to investigate further this phase of evolution.

5   DISCUSSION

Given the results of the previous two sections it is now possible to compare the model predictions with recent X-ray data and to consider the evolution of the radio source and the cluster itself.

5.1   Comparison to X-ray observations

As discussed in the introduction there exist a number of studies of the interaction of radio sources with their environment probed by X-ray data. In this section I will focus on those published studies of interactions which have sufficient resolution and sensitivity to probe directly the gas between bow-shock and contact surface.

The CHANDRA results for the Perseus cluster (Fabian et al. 2000; Fabian et al. 2002) show clear evidence of a radio source displacing the cluster medium. Holes are observed in the X-ray emission spatially coincident with the bright radio

emission from 3C84. Surrounding these holes are bright X-ray rims which are cooler than the surrounding gas. Fabian et al. (2002) conclude that the source must be expanding subsonically since the heating effect of a strong shock is not observed. The results of Section 3 provide an alternative explanation for these data although the explanation proposed by Fabian et al. cannot be ruled out. For supersonic expansion into a declining atmosphere under adiabatic conditions I have shown that a layer of cool bright X-ray gas surrounds the cocoon, although immediately behind the shock the gas temperature is still increased. The observational data will effectively measure an emission weighted temperature and therefore the cool bright rims surrounding the holes can be interpreted as emission from a layer of cool gas adjacent to the contact surface. A similar conclusion applies to the data for Abell 2052/3C317 (Blanton et al. 2001) in which cool X-ray rims are again observed surrounding holes in the X-ray emission spatially coincident with the radio structure. In this case optical emission is also observed from the X-ray rims (Baum et al. 1988) consistent with a cooling time in the swept-up gas which is significantly reduced compared to that in the undisturbed ICM again consistent with the analysis of Section 3. Both of these comparisons should be treated with some caution since the radio structure of both 3C84 and 3C317 is complex and does not resemble a classical FR-II, however the analysis presented here is still applicable if the radio structures are surrounded by a bow shock.

The best test case is that of Cygnus A. In this case the recent CHANDRA results presented by Smith et al. (2002) permit an estimate of the mass of cool gas which may be present. For a hotspot advance speed such as that derived from spectral ageing of order 0.04-0.05c (Alexander et al. 1984), and a ratio of cocoon radius to length of approximately 0.2 - 0.25, the expansion velocity of the cocoon is of order 0.01c. Using the data from Smith et al. at a radius typical of a point mid-way along the cocoon (~ 70kpc) the sound speed in the ICM is of order 0.003c, giving a Mach number for the sideways expansion of the cocoon of about 3. The results of Section 3 suggest that all gas swept up for $y \sim 0.2$ will now be cooler than the gas just ahead of the bow-shock at a cluster radius of ~ 70kpc. Estimating the mass of gas this corresponds to is very uncertain. The linear extent of Cygnus A in the cluster is of order 100kpc and the total gas mass within this radius is or order $10^{12} M_\odot$; within this radius Cygnus occupies about 6% of the volume giving a total mass of swept up gas of order $6 \times 10^{10} M_\odot$. Again, assuming a relatively steep atmosphere this suggests that approximately $10^{10} M_\odot$ of gas swept up by the radio source may have cooled to below the temperature of the ICM. As discussed by KA99, flow between the bow-shock and contact surface will lead to accumulation of gas around the edge of the cocoon nearest to the host galaxy and it is here that the cool gas should therefore be found. The detailed calculation of KA99 when applied to Cygnus A also predicted an annulus of cold gas in this location. Interestingly, the observations of Smith et al. (2002) show conclusive evidence for arcs of cooler gas arranged in a belts around the host galaxy which are naturally explained as resulting from the accumulation of adiabatically cooled swept-up ICM. If the advance speed of the hotspot and cocoon are significantly less than this value as suggested by considering the ram pressure within the hotspot and cocoon (Alexander & Pooley 1996) then

the discussion of Section 4.1 shows that the contact surface may be RT unstable. If this is the case then the swept up gas may well be being entrained within the cocoon. This is most likely to occur in the first instance to the sides of the cocoon where most cold material has accumulated. In this case the belts of cooler X-ray gas will be falling through the contact surface and may, as suggested by by Smith et al. (2002) provide a mechanism for continued fuelling of the AGN.

5.2 Evolution of a radio source

The radio source sweeps up a very significant mass of gas during its lifetime which is displaced from the centre of the cluster. Depending upon the effectiveness of thermal conduction this gas may cool adiabatically to form a thin dense layer of relatively short cooling time (relative to the ambient ICM at a similar distance from the cluster centre).

Both the inertia and gravitational forces exerted by the swept-up gas will modify the dynamics of the radio source. Using the results above it is possible to determine in outline the likely evolution of a powerful source. The initial stage of evolution will be as the source expands through the core of the cluster or the ISM of the host galaxy. In either case the mean density of the atmosphere will be approximately constant. During this phase the source will follow a self-similar form of evolution as discussed by KA97 and Alexander (2000). Although the effects of the swept-up mass are growing, they do not become significant while the source is in this phase (Section 4.2). As shown by KA99 there will be flow of the swept-up gas away from the region ahead of the hotspot to the side of the source. Despite this preferential accumulation of material at the surface of the cocoon self-similar expansion will still occur since there is a feedback between the cocoon pressure and hotspot pressure brought about by the jet reaching pressure balance with the cocoon.

If the source is sufficiently powerful, and the jet is not disrupted by a Kelvin-Helmhotz instability (Alexander 2000), the source will reach a radius equal to the core radius of the atmosphere. The hotspot clearly reaches the region of declining density first, and as the source passes from the cluster core to a declining atmosphere the aspect ratio is likely to increase in a phase of non self-similar expansion. The cocoon pressure will remain high determined by the properties of the atmosphere in the cluster core, the jet pressure will therefore also remain high and hence the hotspot pressure will exceed that predicted for self-similar expansion into a declining atmosphere. This phase will cease when the bow-shock fully enters the declining atmosphere, and the source re-enters a phase of self-similar expansion. Beyond this point the effect of the swept-up gas (Section 4.2) becomes important and as the Mach number of the jet decreases the aspect ratio of the source will again increase.

Throughout this expansion the contact surface may be liable to RT instability. This is most severe when the deceleration of the contact surface is small. During the transition of the source through the cluster core this is likely to be the case. Any instability will allow swept-up gas to be entrained into the cocoon material and since the density of this gas increases towards the centre of the source, it seems likely that any instability will be most apparent in this region. This process is also seen in the simulations of Reynolds et al. (2001). The source may well survive this entrainment if the jet is not disrupted, but this does form a natural mechanism for excluding the cocoon from the region around the host galaxy.

The final stages of the evolution of the source will be dominated by the onset of the RT instability as the source slows. The gravitational forces on the swept-up gas will be largest near the host galaxy (where most of the swept-up gas has accumulated). These forces will lead to a pinching off of the cocoon near the host galaxy and the onset of a buoyant phase (Brügen & Kaiser 2001). It may well be that the ingress of material at this time disrupts the jet and effectively kills the source. The buoyant phase will result in the mixing of the remaining swept-up gas as discussed above or, when the geometry is optimal, the formation of a true mushroom cloud of radio-emitting plasma as discussed by Brügen & Kaiser (2001). An alternative scenario is that the RT instability is effective over the entire contact surface; in this case mixing of the radio emitting plasma and swept-up gas will be very efficient and the cocoon will be filled with X-ray emitting material with the radio-emitting plasma having a non-unity filling factor. Pockets of low-density cocoon material will remain buoyant leading to very efficient mixing.

The analysis of the previous two sections has considered a simple model for the cluster atmosphere and in particular the relationship between the swept-up mass and the depth of the potential well. If the cluster gas were pre-heated the gas pressure in the atmosphere will dominate over gravitational forces of the swept-up layer and the mass of this layer may be negligible. In principle it may be possible to use the dynamics of radio sources to provide an additional probe of the cluster environment, however this requires an independent method of measuring the pressure within the cocoon and expansion speed of the source. The latter may be accessible to VLBI in the future, however at present the equation of state of the radio-emitting plasma is very uncertain (e.g. Hardcastel & Worrall 2000) and this must be resolved before radio sources can be used as detailed probes of their environment.

5.3 The effect on the ICM

It is now possible to consider the effect of the radio source on the ICM. Whether the thermal conductivity is suppressed or not, the radio source must do significant work on the ICM; the total pdV work must be comparable to the stored energy within the lobes (KA97, KA99), and this is deposited as heat within the gas between the bow-shock and contact surface. However, the effect of the radio source is also to move cluster gas out of the centre of the cluster to large radii.

In the early stages of evolution of the radio source we expect high Mach-number expansion into a relatively flat atmosphere. Whether the thermal conductivity is suppressed or not the results derived above show that the swept-up gas is efficiently heated, a bright X-ray shell develops between the cocoon and bow-shock and this gas has a cooling time longer than the surrounding material. Furthermore, the radio source is lifting a large mass of gas out of the centre of the cluster. Clearly during this phase the radio source halts any cooling flow in the cluster.

As the source expands into a declining atmosphere this picture changes. If the gas behaves adiabatically and the

cluster atmosphere is relatively steep, $\beta > 1$, then the effect of the radio source is to deposit a substantial amount of gas at large cluster radii which has a cooling time less than the existing ICM at a similar radius. Just ahead of the contact surface there will a layer of cooler gas. In the model developed above there would apparently always be some gas at arbitrarily low temperature, however this is clearly an artefact of assuming a power-law atmosphere extending to the centre of the cluster. In reality, this layer will be thin, while the source is expanding highly supersonically, however as the Mach number of the expanding cocoon falls the mass of cooler gas will become significant. Even for what we think of as powerful supersonically expanding sources a significant mass of gas may be in a cooler phase.

As the source slows towards sub-sonic expansion the onset of RT instabilities as discussed above will lead to a mixing of the swept-up gas with the ambient gas and the cocoon material. The effect of this process on the cluster may be dramatic. If the swept-up gas has evolved adiabatically and the expansion has been into a steep atmosphere then the effect is to deposit gas at large cluster radii with a cooling time less than the ambient material. Indeed some gas adjacent to the contact surface may be quite cool and have a very short cooling time. It seems likely therefore that the mixing process driven by the buoyancy of the radio plasma will lead to a multi-phase ICM. As the mixing proceeds this multi-phase gas must refill the cavity occupied by the radio source. This will either be dominated by fluid-dynamical processes or dynamical infall of the densest gas. In either case the timescale will be of order the sound crossing time, t_s, across the radius of the cluster out to which the source expanded. The age of the radio source however is of order t_s/M where M is an average Mach number for the expansion. Therefore the timescale for the cluster gas to refill the cocoon must be of order M times the age of the source when it dies; this can clearly be as long as an order of magnitude. During this phase their will be no distinct radio source (a cluster halo source may possibly exist), but there will be a very significant inflow of multi-phase cluster gas. At the end of this period the cluster within at least the radius reached by the source will be filled with a multi-phase medium with a cooling time in the outer parts of this region reduced over that prior to the existence of the radio source.

Similar conclusions concerning the effect of AGN heating were reached by Binney & Tabor (1995); their analysis also predicts suppression of the cooling flow followed by a period of catastrophic cooling after the period of AGN heating. The long term effect of a radio source event on the cluster is however likely to be determined by the buoyant rise of material through the cluster atmosphere (Churazov et al. 2001; Brüggen & Kaiser 2001; Quillis, Bower & Balogh 2001). The subsequent evolution of this (no doubt turbulent) multi-phase medium is difficult to determine without the aid of simulations. Whether a subsequent cooling flow is enhanced or suppressed will be determined not only by the details of this multi-phase ICM, but also by the epoch at which the cluster is observed relative to the last radio-source event.

6 CONCLUSIONS

The effect of a radio source on the cluster gas and vice-versa has been considered. I have shown that, if the thermal conductivity in the gas is suppressed as has been suggested by recent observations with CHANDRA, then a cool dense layer of swept-up ICM material will form around the contact surface of a powerful FR-II radio source expanding into a steeply falling atmosphere. This results simply from the adiabatic expansion of dense gas as it is carried out of the potential well of the cluster ahead of the advancing cocoon of the radio source and only occurs when the density of the cluster atmosphere falls faster than $1/r$ for large cluster radii. As the source slows from highly supersonic expansion to near sub-sonic expansion almost all of the swept-up gas has a cooling time less than the ambient ICM; some gas is likely to be very cool and there is the possibility of promoting the formation of a multi-phase medium.

Eventually the layer of swept-up gas must become Rayleigh-Taylor unstable. The swept-up gas accumulates preferentially at the edges of the cocoon away from the hotspot due to flow in the swept-up layer driven by the pressure gradient which must exist due to the difference in pressure between hotspot and cocoon. Once the instability sets in gas will fall through the contact surface and begin to fill the cavity formed by the cocoon. Using existing simulations as a clue to the behaviour of the source in this phase it would seem that depending on the detailed geometry of the source at this time there will be both infall of material and mixing of the swept-up gas with the ambient ICM.

The long-term effect of the radio source on the cluster when there is (a) suppression of the thermal conductivity and (b) efficient mixing of the swept-up layer with the ambient gas, may well be to reduce the cooling time of the gas at large cluster radii. This may promote the formation of a multi-phase ICM and/or for a period of order a few times the lifetime of the source enhance or instigate a cooling flow. Certainly there must be a substantial flow back into the cluster core as the swept-up mass falls through the contact surface and fills the cocoon.

The swept-up ICM must also have a significant effect on the dynamics of the radio source especially as the source slows towards sub-sonic expansion; if there is no pre-heating of the cluster gas then the gravitational force of the swept-up gas per unit area of the cocoon must be at least comparable to the external pressure. For those sources which escape the core radius of the cluster the gravitational force due to the swept-up gas can lead to an increase in the aspect ratio of the source as the Mach number of expansion decreases. For very steep atmospheres falling nearly as fast as $1/r^2$ this perturbation does not grow and the source can expand throughout its life in a self-similar fashion.

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APPENDIX A: THE DYNAMICAL MODEL

In this appendix I give more details of the solution to the dynamical equations governing the evolution of the cocoon. The three governing equations in dimensionless form are (see Section 4.2):

$$3\lambda^3 f^3 \frac{d}{d\tau}\left(\zeta \dot{f}^2 f^{-\beta}\right) + 4\zeta \dot{f}^2 f^{-\beta}\frac{d}{d\tau}\left(\lambda^3 f^3\right) = \frac{1}{\pi} \quad (A1)$$

$$\frac{3}{2}(1-\lambda^3)f^3\frac{d}{d\tau}\left(\dot{f}^2 f^{-\beta}\right) + \frac{5}{2}\dot{f}^2 f^{-\beta}\frac{d}{d\tau}\left((1-\lambda^3)f^3\right) =$$
$$\frac{15}{8}f^{2-\beta}\dot{f}^3 \quad (A2)$$

$$\frac{3(3-\beta)}{4}\lambda^2 f^{2-\beta}\dot{f}^2 = f^{3-\beta}\left(\frac{d^2}{d\tau^2}(\lambda f) + Y_D(\lambda f)^{\beta_D}\right) \quad (A3)$$

where $Y_D = t_0^2 X_D/a_0$ $\beta_D = \pm 1$ and a dot indicates differentiation with respect to τ. The third equation as written assumes $\beta > 1$; for $\beta < 1$ the equivalent equation is found by replacing each occurrence of λf by f.

These equations admit a self-similar solution when the gravitational term is negligible $Y_D \approx 0$ with λ and ζ equal to constant values (λ_0, ζ_0) and $f = C\tau^\delta$. Substituting for this form for f in A1 gives

$$\frac{1}{\pi C^{(5-\beta)}\lambda_0^3\zeta_0} = \frac{27}{(5-\beta)^3}(8-\beta)$$

and from A2:

$$1 - \lambda_0^3 = \frac{15}{4(11-\beta)}$$

and from A3:

$$\zeta_0 = 1 - \left(\frac{1-\delta}{\delta}\right)\frac{4}{3(3-\beta)\lambda_0}$$

and $\delta = 3/(5-\beta)$. This result again holds for $\beta > 1$, for $\beta < 1$:

$$\zeta_0 = 1 - \left(\frac{1-\delta}{\delta}\right)\frac{4}{3(3-\beta)}$$

When Y_D is small but non-zero, I seek a perturbed solution of the form $f = f_0(\tau)(1 + F)$, $\lambda^3 = \lambda_0^3(1 + L)$ and $\zeta = \zeta_0(1 + Z)$, where $f_0(\tau) = C\tau^{3/(5-\beta)}$. Substituting these trial solutions into A1 to A3 and neglecting any terms which are quadratic or higher powers of F, L, Z, $\dot{F}$, $\ddot{F}$, $\dot{L}$, $\ddot{L}$, or Y_D gives:

$$0 = 3[(2-\beta)\delta - 2]\left(L + Z + (5-\beta)F + \frac{2\tau}{\delta}\dot{F}\right) +$$
$$3\tau\left(\dot{Z} + (2-\beta)\dot{F} + \frac{2}{\delta}\dot{F} + \frac{2\tau}{\delta}\ddot{F}\right) +$$
$$12\delta\left(Z + L + (5-\beta)F + \frac{2\tau}{\delta}\dot{F}\right) +$$
$$4\tau\left(\dot{L} + 3\dot{F}\right) \quad (A4)$$

$$\frac{15}{8}\delta\left((5-\beta)F + \frac{3\tau}{\delta}\dot{F}\right) =$$
$$\frac{3}{2}\lambda_3[(2-\beta)\delta + 2]\left((5-\beta)F - l + \frac{2\tau}{\delta}\dot{F}\right) +$$
$$\frac{3\tau}{2}\lambda_3\left((2-\beta)\dot{F} + \frac{2}{\delta}\dot{F} + \frac{2\tau}{\delta}\ddot{F}\right) +$$
$$\frac{15\delta}{2}\lambda_3\left((5-\beta)F - l + \frac{2\tau}{\delta}\dot{F}\right) +$$
$$\frac{5\tau}{2}\lambda_3\left(3\dot{F} - \dot{l}\right) \quad (A5)$$

where $\lambda_3 = 1 - \lambda_0^3$ and $l = L/\lambda_3$. Equation A3 becomes

$$\delta(\delta - 1)\left(\frac{Z}{\zeta_0 - 1} + \frac{2\tau}{\delta}\dot{F} - \frac{L}{3}\right) =$$

$$2\tau\delta\left(\dot{F}+\frac{\dot{L}}{3}\right)+\tau^2\left(\ddot{F}+\frac{\ddot{L}}{3}\right)+Y_D(\lambda_0 f_0)^{\beta_D-1} \quad \text{(A6)}$$

for $\beta > 1$, and where $f_0 = Ca_0\tau^{3/(5-\beta)}$. For $\beta < 1$ the terms involving L, $\dot{L}$ and $\ddot{L}$ do not appear and $\lambda_0 f_0$ becomes simply f_0. A solution then exists such that each term (F, L, Z) has a power-law dependence on dimensionless time of the form τ^α, and this time dependence is the same as the perturbation, $\tau^2 Y_D(\lambda_0 f_0)^{\beta_D-1}$, giving $\alpha = 2 + 3(\beta_D - 1)/(5 - \beta)$.

Substituting this form of the solution with $F = F_1\tau^\alpha$ etc. we obtain a set of algebraic equations for the amplitudes F_1, L_1 and K_1.

$$[(18 - 3\beta)\delta - 6 + 3\alpha]\left(Z_1 + L_1 + (5 - \beta)F_1 + \frac{2\alpha}{\delta}F_1\right)$$
$$+ \alpha\left(L_1 + 3F_1\right) = 0 \quad \text{(A7)}$$

$$\frac{15\delta}{4\lambda_3}\left[(5 - \beta) + \frac{3\alpha}{\delta}\right]F_1 = 2\alpha\left(3F_1 - l_1\right) +$$
$$3[\alpha + (7 - \beta)\delta - 2]\left((5 - \beta)F_1 - l_1 + \frac{2\alpha}{\delta}F_1\right) \quad \text{(A8)}$$

$$\delta(\delta - 1)\left(\frac{Z_1}{\zeta_0 - 1} + \frac{2\alpha F_1}{\delta} - \frac{L_1}{3}\right) =$$
$$[2\alpha\delta + \alpha(\alpha - 1)]\left(F_1 + \frac{L_1}{3}\right) + Y_D\lambda_0^{\beta_D-1}C^{\beta_D-1} \quad \text{(A9)}$$

Again equation A9 is written for $\beta > 1$ and the equivalent form for $\beta < 1$ has no terms in L_1 and $\lambda_0^{\beta_D-1}$ is replaced by unity. Given values for β and β_D, the solution of these equations is elementary.